2-Factorization Theorem in Finite Groups (有限群論)

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2-factorization theorem in finite groups

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1. 2-factorization in finite solvable groups

Let π be a set of primes which contains 2.

Hypothesis A. Let S be a non-identity finite 2-group and G a finite (solvable) group.

(A.1) S is a Sylow 2-subgroup of G.

(A.2) O(G) = 1.

(A.3) G is a π-group.

Question 1. Let S be a non-identity finite 2-group. Then does there exists a non-identity characteristic subgroup W(S) of S which is normal in G, for any finite (solvable) group G which satisfies Hypothesis A ?

If π contains 3, there exists the following counter example group of Question 1 ; the semi-direct product group of a four group and SL(2,2), i.e. a symmetric group of degree 4.

G. Glauberman proposed the question 1 as above, in the case π = {p ;

prime $\neq 3\}$, in his paper [2]. In the case S is a p-group, p odd, this question had been solved by himself[1] so called ZJ-theorem, furthermore he showed a counter example group of ZJ-theorem in the case $p = 2$.

In the case $p = 2$, the almost only one result is one of J. G. Thompson [4], which is obtained until now. And he proposed an interesting conjecture [5]. After the congress in Kyoto, the autor may obtain the following result ;

Theorem [3]. In the case $\pi = \{2, p\ ;\ 2^m \not\equiv -1 \pmod{p},\ m = 1, 2, \cdots, p-1\}$, Question 1 is affirmative.

2. 2-factorization in finite non-solvable group

Hypothesis B. Let S be a finite 2-group and G a finite (non-solvable) group.

(B.1) S is a Sylow 2-subgroup of G.

(B.2) $C_G(O_2(G)) \subseteq O_2(G)$.

Question 2. Assume that S and G satisfy Hypothesis B. If $G \neq N_G(J(S))C_G(Z(S))$, we may say about G or $O_2(G)$?

If $G/O_2(G)$ is a simple group of Lie type of characteristic odd type, (in the half case) we have $G = N_G(J(S))C_G(Z(S))$.

Question 3. Assume that S and G satisfy Hypothesis B. Set $\bar{G} = G/\Phi(O_2(G))$ and $V = O_2(G)/\Phi(O_2(G)) = V_1 \times \cdots \times V_r \times C_V(G)$, where V_i is an $\bar{G}$-irreducible component of V on which $G/O_2(G)$ acts faithfully. (Note that we may not always set as this !) Suppose that $r \geq 2$. Then $G = N_G(J(S))C_G(Z(S))$?

References

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[5] J. G. Thompson, Replacement theorem for finite groups and a conjecture, J. Algebra 13 (1969), 149-151.